BirZeit University
Faculty of Science-Department of Physics
Quantum Mechanics Phys635
Spring 2016
Final Exam, May. 30th 2016

1. (10 points) If $|i>$ and $|j>$ are eigenkets of Hermitian operator A. Under what conditions is $(|i> + |j>)$ an eigenket of A

2. (15 points) Show that any operator that commutes with two Cartesian components of the angular momentum operator necessarily commutes with the total angular momentum operator.

3. Define $N = b^\dagger b$, where $b^\dagger$ being the Hermitian conjugate of the operator **b**, where $b^\dagger b + bb^\dagger = 1$, and $b^2 = 0$.
   (a) (10 points) Is **b** Hermitian?
   (b) (5 points) Is **N** Hermitian?
   (c) (10 points) Show that $N^2 = N$ and find the eigenvalues of **N**.

4. Let $\vec{s}_1$ and $\vec{s}_2$ be the spin operators of two spin 1/2-particles. Then $\vec{S} = \vec{s}_1 + \vec{s}_2$ is the spin operator for this two-particle system.
   (a) (10 points) Consider the Hamiltonian $H_0 = \frac{1}{\hbar^2}(S_x^2 + S_y^2 - S_z^2)$. Determine the eigenvalues and eigenvectors of this Hamiltonian.
   (b) (20 points) Consider the perturbation $H_1 = s_{1x} - s_{2x}$. Calculate the eigenvalues of $H_0 + \lambda H_1$ in first-order perturbation theory.

5. Suppose that a hydrogen atom is exposed to a uniform electric field, $\vec{\varepsilon}$, and a parallel, uniform magnetic field, $\vec{B}$. Consider the first excited energy level, corresponding to n = 2.
   (a) (15 points) Show that in general the level is split into four nondegenerate energy levels.
   (b) (10 points) For what values of ε and B are there instead only three levels, and what are the degeneracies of these levels?
   (c) (10 points) For what values of ε and B are there only two levels, and what are the degeneracies of these levels?

6. (25 points) Consider a particle bound in a simple harmonic oscillator potential. Initially(t <0), it is in the ground state. At t = 0 a perturbation of the form

$$H_0(x;t) = Ax^2 e^{-\frac{t}{\tau}}$$

   is switched on. Using time-dependent perturbation theory, calculate the probability that, after a sufficiently long time ($t >> \tau$), the system will have made a transition to a given excited state. Consider all the states.

7. Consider two coupled oscillators A and B described by annihilation/creation operators $\hat{a}$, $\hat{a}^\dagger$ and $\hat{b}$, $\hat{b}^\dagger$ respectively, satisfying the usual commutation relations. The Hamiltonian describing their dynamics is given to be:

$$\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + \hat{b}^\dagger\hat{b} + 1) + \hbar k(\hat{a}\hat{b}^\dagger + \hat{b}\hat{a}^\dagger)$$

   where k is the "coupling constant".
   (a) (15 points) Show that the solution to the Heisenberg equations of motion are:

$$\hat{a}(t) = e^{-i\omega t}(\hat{a}(0)cos(kt) - i\hat{b}(0)sin(kt))$$
$$\hat{b}(t) = e^{-i\omega t}(\hat{b}(0)cos(kt) - i\hat{a}(0)sin(kt))$$

(b) (15 points) Suppose that the state of the system (in the Schrödinger picture) at t=0 is a product state consisting of oscillator A in the first excited state and B in the ground state, $|\psi(0)>=|1_A> \times |0_B>$. Find the state of the composite system at later times, and show that the reduced density operator for oscillator A alone in the basis $|1_A>, |0_A>$ is:

$$\hat{\rho}_A = \begin{pmatrix} cos^2(kt) & 0 \\ 0 & sin^2(kt) \end{pmatrix}$$

(c) (5 points) How does the "purity" of the state $\hat{\rho}_A$ vary with time?

Good Luck

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Question:	1	2	3	4	5	6	7	Total
Points:	10	15	25	30	35	25	35	175
Score:								